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On the Exact Travelling wave solution of the PHI-Four equation

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ABSTRACT
In this study, we established a traveling wave solution by using ansatz method say, Sine-function method for the PHI-Four equation.
Keywords: Sine-function method, PHI-Four equation, solution solutions.

I. INTRODUCTION

The searching for explicit exact solutions of nonlinear partial differential equations (NLPDE) plays an important role in the study of nonlinear physical phenomena. In past few decades, a number of very powerful and direct methods have been proposed and developed to find the explicit solutions of NLPDEs, such as the tanh method [1,2], the extended tanh-function method [3,4], the modified extended tanh-function method [5-9], variational iteration method [10-15], the generalized hyperbolic-function [16,17], the separation of variables method [18.19], First Integral Method [20,21] and so on.

The purpose of this paper is to extend the sine function method to find the exact soliton solutions of the important nonlinear partial differential equation PHI-Four equation.

II. THE SINE FUNCTION METHOD

Consider the nonlinear partial differential equation of the form

$$F(u, u_t, u_x, u_{xx}, u_{xxt}, \ldots) = 0$$
(1)

where $u(x,t)$ is the solution of nonlinear partial differential equation (1). We use the transformations,

$$u(x,t) = f(\xi), \quad \xi = x - ct$$
(2)

This enables us to use the following changes:

$$\frac{\partial}{\partial t}(\cdot) = -c\frac{d}{d\,\xi}(\cdot), \quad \frac{\partial}{\partial x}(\cdot) = \frac{d}{d\,\xi}(\cdot), \quad \frac{\partial^2}{\partial x^2}(\cdot) = \frac{d^2}{d\,\xi^2}(\cdot), \ldots$$
(3)

Eq. (3) changes Eq. (1) in the form

$$G(f, f', f'', f''', \ldots) = 0$$
(4)

The solution of Eq. (4) can be expressed in the form:

$$f(\xi) = \lambda \sin^{\alpha}(\mu\,\xi), \qquad |\xi| \le \frac{\pi}{\mu}$$
(5)

where λ, α and μ are unknown parameters which are to be determined. Thus we have:

$$f' = \frac{df(\xi)}{d\xi} = \lambda\,\alpha\,\mu\,\sin^{\alpha-1}(\mu\,\xi)\cos(\mu\,\xi)$$
(6)

$$f'' = \frac{d^2 f(\xi)}{d\xi^2} = -\lambda\,\mu^2\alpha\,\sin^{\alpha}(\mu\xi) + \lambda\,\mu^2\alpha(\alpha - 1 \sin\alpha - 2\mu\xi - \lambda\,\mu 2\alpha(\alpha-1)\sin\alpha(\mu\xi)$$
(7)

Substituting Eq. (5) in Eq. (4) gives a trigonometric equation of $\sin^{\alpha}(\mu\,\xi)$ terms. To determine the parameters first balancing the exponents of each pair of sine to find α. Then collecting all terms with the same power in $\sin^{\alpha}(\mu\,\xi)$ and put to zero their coefficients to get a system of algebraic equations among the unknowns λ, α and μ. Now the problem is reduced to a system of algebraic equations that can be solved to obtain the unknown parameters λ, α and μ. Hence, the solution considered in Eq. (5) is obtained. The above analysis yields the following theorem:

Theorem: The exact analytical solution of the nonlinear partial differential equations Eq. (1) can

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be determined in the form Eq. (5) where all constants found from the algebraic equations after its solutions.

III. APPLICATION

In order to illustrate the effectiveness of the proposed method we will consider the PHI-Four equation [22] in the following form:

$$u_{tt} - au_{xx} - \varepsilon u + bu^3 = 0 \tag{8}$$

Using the transformation, $u(x,t) = f(\xi)$, $\xi = x - ct$, Eq. (8) reduces to:

$$(c^2 - a)f'' - \varepsilon f + bf^3 = 0 \tag{9}$$

Substituting Eq. (5) and (7) into (9) gives:

$$-(c^2-a)\lambda\alpha\mu^2 \sin^{\alpha}(\mu\,\xi) + (c^2-a)\lambda\mu^2\alpha(\alpha-1)\sin^{\alpha-2}(\mu\,\xi) - (c^2-a)\lambda\mu^2\alpha(\alpha-1)\sin^{\alpha}(\mu\,\xi) - \varepsilon\lambda\sin^{\alpha}(\mu\,\xi) + b\lambda^3\sin^{3\alpha}(\mu\,\xi) = 0 \tag{10}$$

Eq. (10) is satisfied only if the following system of algebraic equations holds:

$$\begin{aligned} &3\alpha = \alpha - 2,\\ &-(c^2-a)\lambda\alpha\mu^2 - (c^2-a)\lambda\mu^2\alpha(\alpha-1) - \varepsilon\lambda = 0,\\ &b\lambda^3 + (c^2-a)\lambda\mu^2\alpha(\alpha-1) = 0. \end{aligned} \tag{11}$$

solving the system of equations (11), we obtain:

$$\alpha = -1, \quad \mu = \pm\sqrt{\frac{\varepsilon}{(a-c^2)}}, \quad \text{and} \quad \lambda = \pm\sqrt{\frac{2\varepsilon}{b}} \tag{12}$$

substituting Eq. (13) into Eq. (5) we obtain the exact soliton solution of the PHI-Four equation

$$u(x,t) = \pm\sqrt{\frac{2\varepsilon}{b}}\,\sin^{-1}\left(\pm\sqrt{\frac{\varepsilon}{(a-c^2)}}\,(x-ct)\right) \tag{13}$$

This gives the desired exact soliton solution of the PHI-Four equation.

IV. CONCLUSION

In this paper, the ansatz method, sine-function method has been successfully applied to find the solution for PHI-Four equation. The sine-function method is used to find new exact solution. Thus, it is possible that the proposed method can be extended to solve the problems of nonlinear partial differential equations which arising in the theory of solitons and other areas.

REFERENCES

[1]. E.J. Parkes, B.R. Duffy, Comput. Phys. Commun. 98 (1998) 288.
[2]. A.H. Khater, W. Malfiet, D.K. Callebaut, E.S. Kamel, Chaos Solitons Fractals 14 (2002) 513.
[3]. E. Fan, Phys. Lett. A 277 (2000) 212.
[4]. E. Fan, Z. Naturforsch. A 56 (2001) 312.
[5]. S.A. Elwakil, S.K. El-Labany, M.A. Zahran, R. Sabry, Phys. Lett. A 299 (2002) 179.
[6]. S.A. EL-Wakil, M.A. Abdou, Chaos Solitons Fractals 31 (2007) 840.
[7]. S.A. EL-Wakil, M.A. Abdou, Chaos Solitons Fractals 31 (2007) 1256.
[8]. A.A. Soliman, Physica A 361 (2) (2006) 394.
[9]. M.A. Abdou, A.A. Soliman, Phys. Lett. A 353 (2006) 487.
[10]. M.A. Abdou, A.A. Soliman, J. Comput. Appl. Math. 181 (2) (2005) 245.
[11]. A.A. Soliman, Math. Comput. Simul. 70 (2) (2005) 119.
[12]. M.A. Abdou, A.A. Soliman, Physica D 211 (2005) 1.
[13]. A.A. Soliman, Chaos Solitons Fractals 29 (2006) 294.
[14]. A.A. Soliman, M.A. Abdou, Numerical solutions of nonlinear evolution equations using variational iteration method, J. Comput. Appl. Math. (2007).
[15]. M.A. Abdou, On the variational iteration method, Phys. Lett. A (2007).
[16]. Y.T. Gao, B. Tian, Comput. Phys. Commun. 133 (2001) 158.
[17]. B. Tian, Y.T. Gao, Z. Naturforsch. A 57 (2002) 39.
[18]. X.-Y. Tang, S.-Y. Lou, Chaos Solitons Fractals 14 (2002) 1451.
[19]. X.-Y. Tang, S.-Y. Lou, Phys. Rev. E 66 (2002) 46601.

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[20]. P. Sharma and O.Y. Kushel, International Journal of Nonlinear Science 10 (1) (2010) 46.
[21]. V.G. Gupta and P. Sharma, Ganita Sandesh 24 (1) (2010) 21.
[22]. J. Cao, M. Song and A. Biswas, Bull. Malays. Sci.Soc. (2) 37 (4) (2014) 1209.